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The surface $OA'A$ is the total conical surface less the ungula, that is,

$$\frac{\pi}{2}\sin\alpha[2K'^2-2K'^2+(K+K')\sqrt{KK'}]=\frac{\pi}{2}(K+K')(KK')^{\frac{1}{2}}\sin\alpha.$$

The problem rests upon the tedious integration of the form,

$$\int_r^R 2x\cos^{-1}\left[\frac{2Rr-(R+r)x}{(R-r)x}\right]dx.$$

Also solved by A. M. Harding and P. Penalver.

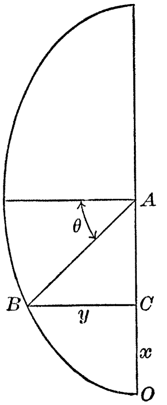
A solution of 334 was received from Elmer Schuyler, after the the forms were sent to the printer.

MECHANICS.

270. Proposed by W. J. GREENSTREET, Burghfield, England.

A cycloid has its base vertical. Find the line of quickest descent from the middle point of the base, and its approximate inclination to the horizon.

Solution by J. Scheffer, Hagerstown, Md.



Let AB be any straight line drawn from the middle point A of base of the cycloid, and let θ be the angle that it makes with the horizon. The time of descent from A to B is $t=\sqrt{\frac{2}{g}\cdot\frac{s}{\sin\theta}}$, AB being $=s$. But $s^2=(r\pi-x)^2+y^2$, x and y being subject to the equations of the cycloid, $x=r(\phi-\sin\phi)$, $y=r(1-\cos\phi)$. Substituting, we have $s^2=r^2[(\pi-\phi)+\sin\phi]^2+(1-\cos\phi)^2$. We have to get the minimum of $s/\sin\theta$, which reduces to

$$\frac{(\psi+\sin\psi)^2+(1+\cos\psi)^2}{\psi+\sin\psi},$$

after putting $\pi-\phi=\psi$ and omitting the constant r^2.

Differentiating and setting the differential coefficient equal to zero, we get, after some easy reductions, the transcendental equation $\psi^2-2\cos\psi-2=0$. Whence $\psi=\pm 2\cos\psi/2$, and using the positive sign, we find $\psi=85°$, nearly. Then

$$\tan\theta=\frac{r\pi-x}{y}=\frac{\psi+\sin\psi}{1+\cos\psi}.$$

Hence, finally, $\theta=53°\ 39'$, nearly.

Note. It is assumed in the above solution that the line sought is the *straight line* of quickest descent. Otherwise the problem becomes much more difficult, requiring for its solution the calculus of variations. The Editors.

NUMBER THEORY.

A solution of 188 was received from Elmer Schuyler after the December issue had been made up.

194. Proposed by L. E. DICKSON, University of Chicago.

Find two numbers a and b each of two digits such that, when the product ab is found by the usual method, the two partial products to be added are exactly the same as the partial products in getting the product ba. Is there a similar pair of numbers of three digits?

SOLUTION BY B. F. YANNEY, University of Wooster.

Let $a = 10x_2 + x_1$, $b = 10y_2 + y_1$.

Then equating corresponding partial products of ab and ba, we get, after reducing, $x_1/y_1 = x_2/y_2$ in each case. Hence this is a necessary condition. Obviously, it is also a sufficient condition.

Examples: $a = 21,\ 32,\ 43;$
$b = 84,\ 96,\ 86.$

In the case of numbers of three digits, let

$$a = 10^2x_3 + 10x_2 + x_1, \quad b = 10^2y_3 + 10y_2 + y_1.$$

Again equating corresponding partial products and reducing, we get

$$10\,x_3y_1 + x_2y_1 = 10\,x_1y_3 + x_1y_2, \tag{1}$$

$$10^2x_3y_2 + x_1y_2 = 10^2x_2y_3 + x_2y_1, \tag{2}$$

$$10\,x_2y_3 + x_1y_3 = 10x_3y_2 + x_3y_1, \tag{3}$$

Since all the digits are less than 10, it is obvious in (2) that $x_3y_2 = x_2y_3$ and $x_1y_2 = x_2y_1$. Whence we get

$$\frac{x_1}{y_1} = \frac{x_2}{y_2} = \frac{x_3}{y_3}. \tag{A}$$

Furthermore, (A) substituted in (1) and (3) gives identities as results. Hence (A) is a necessary condition for the numbers a and b to be such as are required. Obviously also (A) is a sufficient condition. Example: $a = 214$, $b = 428$.

In conclusion we state a general theorem, suggested by the above considerations, the proof of which is not difficult.

THEOREM. Two numbers a and b of n digits each are such that, when the products ab and ba are found by the usual method, the corresponding partial products to be added are exactly the same, if and only if the corresponding digits of the two numbers are proportional.

Illustration:

$$\begin{array}{rcr}
1342 & & 2684 \\
\underline{2684} & & \underline{1342} \\
5368 & = & 5368 \\
10736 & = & 10736 \\
8052 & = & 8052 \\
2684 & = & 2684
\end{array}$$

Solved in similar manner by H. C. FEEMSTER, S. W. REAVES, W. C. EELLS, C. E. GITHENS, A. H. HOLMES, and V. M. SPUNAR.